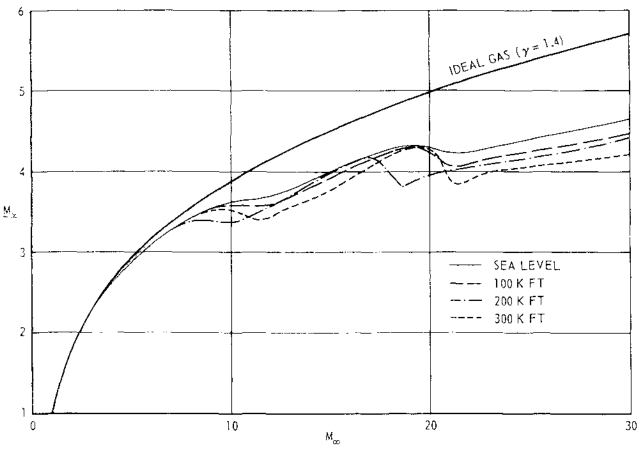

Fig. 1 Asymptotic Mach number for blunt bodies.

atmospheric pressure. Because the flow wetting the body or making up the wake passes through the normal part of the bow shock, a large entropy increase occurs. The static temperature far downstream will then be higher than atmospheric static temperature, and the local velocity will be less than freestream velocity. Thus, the local Mach numbers will be lower than the freestream values.

The asymptotic values of these local "entropy layer" Mach numbers can be determined without knowing the details of the flowfield. Across the shock at the freestream Mach number M_{∞}, the energy equation holds

$$(u_{\infty}^2/2) + h_{\infty} = (u_2^2/2) + h_2 = \text{const} \quad (1)$$

Because of the increase in entropy, h_2 is greater than h_1, and therefore u_2 is less than u_{∞}, whereas from the shock-wave solution, P_2 is greater than P_{∞}. By applying the energy equation between the point just behind the shock and far downstream, where $P_{-\infty} = P_{\infty}$, and making use of the fact that $(S/R)_{-\infty} = (S/R)_2$, the asymptotic velocity $u_{-\infty}$ and asymptotic speed of sound $a_{-\infty}$ are determined. We then have

$$M_{-\infty} = u_{-\infty}/a_{-\infty} \quad (2)$$

This asymptotic Mach number is then useful in checking numerical and theoretical results in regions where the local pressure has nearly reached the ambient pressure.

Figure 1 shows the variation of this asymptotic Mach number with freestream Mach number for several altitudes. The atmosphere used was the 1962 standard,[1] and the local velocity and speed of sound were evaluated by means of a computer program for real-gas normal shocks, coupled to a small program that uses the energy equation to determine the asymptotic velocity. The thermodynamic properties were determined by Hansen's method.[2] The first dip in each real-gas curve in Fig. 1 corresponds to completion of the dissociation of the oxygen molecules; the second corresponds to the completion of the dissociation of the nitrogen molecules. For comparison, the ideal gas curve for $\gamma = 1.4$ is included. Note that the first differences between ideal- and real-gas data appear at Mach numbers between 3 and 4.

The data given in Fig. 1 should provide a rapid check on the operation of both equilibrium and nonequilibrium programs in the far downstream region. Significant deviations from these results are indicative of a possible programing error.

References

[1] "U. S. Standard Atmosphere, 1962," U. S. Committee on Extension to Standard Atmospheres, NASA, U. S. Air Force, and U. S. Weather Bureau (1962).

[2] Hansen, C. F., "Approximations for the thermodynamic and transport properties of high-temperature air," NASA TR R-50 (1959).

Comments on "Criteria for Self-Similar Solutions to Radiative Flow"

K. C. Wang*
Martin Company, Baltimore, Md.

In a recent note,[1] Der discussed the similarity criteria for a radiating piston-type problem in the thin and thick limits. This writer wishes to point out that a generalized version of this problem with the thin and thick limits as special cases has been published previously elsewhere.[2] Der's results can be obtained from Eqs. (14a) and (15) of Ref. 2 by setting $\omega = 0$ which corresponds to the case of ambiant density being constant. The symbol n denotes the same in both papers, but β of Ref. (2) corresponds to α in Der's thin criterion and $-\alpha$ in his thick criterion. This is because Der assumed the absorption coefficient as being proportional to $p^{\alpha}\rho^{\beta}$ for the thin case and to $p^{-\alpha}\rho^{-\beta}$ for the thick case, whereas this writer considered the same as being proportional to $\rho^{\alpha}T^{\beta}$ for arbitrary opacity including the thin and thick limits.

References

[1] Der, J. J., "Criteria for self-similar solutions to radiative flow," AIAA J. **4**, 1107–1108 (1966).

[2] Wang, K. C., "The piston problem with thermal radiation," J. Fluid Mech. **20**, 447–455 (1964).

* Research Scientist. Member AIAA.

Comment on "On Dynamic Snap Buckling of Shallow Arches"

E. H. Dowell*
Princeton University, Princeton, N. J.

An interesting analysis of the title problem using Galerkin's method has been presented by Humphreys. The mathematical and computational features of the analysis are similar to those which arise in several problems concerning the dynamic behavior of plates and shells. Here we wish to discuss briefly the choice of modal functions used in the Galerkin expansion ϕ_m and the evaluation of the integrals of these functions and their derivatives which arise in the analysis.

In Ref. 1, the infinitely wide (or two-dimensional) arch has been treated using the natural modes of simply supported and clamped beams for the ϕ_m. Humphreys notes that although the integrals of ϕ_m for the simply supported case are handled easily, those for the clamped case are difficult to evaluate numerically because of the problem of small differences between large numbers. It is of interest thus to point out that these integrals have been evaluated analytically by Gallagher and Mercer[2] and also by Ketter.[3,4] Gallagher and Mercer do not indicate their method; Ketter, however, has used a clever variation of integration by parts.

The other point to be made is with regard to the finite width (or three-dimensional) problem. Dowell[5] has carried out such an analysis for the simply supported plate (the extension from the plate to a shallow shell is mathematically trivial though physically important) in a different physical

Received July 5, 1966.
* Assistant Professor, Department of Aerospace and Mechanical Sciences. Member AIAA.